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Designer black holes from new 2D gravity

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Abstract

We present a family of extensions of spherically symmetric Einstein-Lanczos-Lovelock gravity. The field equations are second order and obey a generalized Birkhoff's theorem. The Hamiltonian constraint can be written in terms of a generalized Misner-Sharp mass function that determines the form of the vacuum solution. The action can be designed to yield interesting non-singular black-hole spacetimes as the unique vacuum solutions, including the Hayward black hole as well as a completely new one. The new theories therefore provide a consistent starting point for studying the formation and evaporation of non-singular black holes as a possible resolution to the black hole information loss conundrum.

1 Introduction

There exists considerable evidence for the existence of black holes in binary systems and at the center of most galaxies, including our own. This observational evidence makes it imperative to resolve the theoretical puzzles surrounding black holes, most notable among them the so-called information loss conundrum. Interest in information loss has recently been vigorously renewed by the work of Almheiri, Marolf, Polchinski and Sully (AMPS) [1] who argued that the most conservative resolution to the conundrum is the existence near black-hole horizons of a firewall that breaks the quantum correlations between objects on the interior and those on the exterior. AMPS met with considerable resistance to their proposal due in large part to its implied violation of the strong equivalence principle near the horizon. Such quantum-gravity based violations are puzzling given that they must occur, for example, at the horizon of the recently observed [2] ten billion solar mass black hole where the curvature is a hundred orders of magnitude less than the Planck curvature.

It has been known for some time [3, 4, 5, 6] that the information loss problem could in principle be avoided if the singularity were replaced by a regular repulsive core. In this case the result of collapse and evaporation would be a complete spacetime containing a compact trapping horizon as opposed to an event horizon. This possibility was reiterated in a modern context in [7] and put into concrete form by Hayward [8]. More recently it has been actively discussed as an alternative to the firewall proposal [9, 10, 11, 12].

There is substantial literature on static, non-singular black-hole spacetimes [13, 14, 15, 16]. One particular metric that has been studied in some detail [8, 12] is the Hayward metric, whose n -dimensional generalization is

$$ds_{(n)}^2 = - \left(1 - \frac{l^{n-2} M R^2}{R^{n-1} + l^n M} \right) dt^2 + \left(1 - \frac{l^{n-2} M R^2}{R^{n-1} + l^n M} \right)^{-1} dR^2 + R^2 d\Omega_{(n-2)}^2, \quad (1.1)$$

where $d\Omega_{(n-2)}^2$ is the line-element on the unit $(n-2)$ -sphere and we have defined a parameter l that is proportional to the Planck length. The Hayward metric has a non-singular de Sitter core and curvature that is bounded above by $1/l^2$ for arbitrarily large mass¹.

In order to determine whether removing the singularity can solve the information loss problem, it is necessary to study quantitatively the dynamics of non-singular black-hole formation and evaporation. For example, the information must emerge from the black hole before the Bekenstein entropy bound [17] is saturated, which would occur roughly in the so-called Page time [18] when half the mass of the black hole has evaporated via the Hawking process. Most investigations patch local regions to construct dynamical non-singular spacetimes that model the evaporation process [9, 10, 11, 12]. More realistic models should be based on solutions to dynamical equations derived from a diffeomorphism invariant action that includes the back-reaction due to Hawking radiation. The essential

¹GK is grateful to Valeri Frolov for emphasizing the importance of this criterion.

features of this process likely reside in the spherically symmetric sector, so that one can hope to learn much by studying dimensionally reduced, effective two-dimensional (2D) actions. This was done to great effect in the 1990's in the context of 2D dilaton gravity [19] and has been revived more recently in [20, 21, 22]. Until now, however, it has not been possible to use 2D gravity to construct realistic non-singular black holes such as (1.1) with bounded curvature.

The purpose of this Letter is to present a new class of 2D gravity models that are obtained by extending spherically symmetric Einstein-Lanczos-Lovelock gravity in much the same way that 2D dilaton gravity extends general relativity. The resulting theories have all the desirable properties of 2D dilaton gravity, including the existence of a generalized Misner-Sharp mass function that renders the theories classically solvable. The corresponding one parameter family of solutions possess at least one Killing vector. The new actions describe a much larger class of spherically symmetric black-hole spacetimes than previously available, including those for which the maximum curvature is bounded by the Planck scale. They therefore provide a consistent starting point for studying the formation and evaporation of non-singular black holes as a possible alternative resolution to the black hole information conundrum. In this Letter, we adopt units such that $c = \hbar = 1$.

2 The action

The action for the classes of theories we wish to consider is an extension of the spherically symmetric Einstein-Lanczos-Lovelock (EL) gravity [23, 24]. The EL action in $n(\geq 4)$ -dimensional vacuum spacetime is a sum of higher curvature terms.

Consider the spherically symmetric metric in $n \geq 4$ dimensions:

$$ds_{(n)}^2 = \bar{g}_{AB}(y)dy^A dy^B + R^2(y)d\Omega_{(n-2)}^2, \quad (2.1)$$

where $\bar{g}_{AB}(y)$ ($A, B = 0, 1$) is the general two-dimensional Lorentzian metric and R is the areal radius. It was shown in [25, 26, 27] that the dimensionally reduced spherically symmetric EL action can be written as

$$I_L = \frac{\mathcal{A}_{(n-2)}}{16\pi G_{(n)}} \int d^2y \sqrt{-\bar{g}} R^{n-2} \sum_{p=0}^{[n/2]} \alpha_{(p)} \mathcal{L}_{(p)}, \quad (2.2)$$

where $\mathcal{A}_{(n-2)}$ denotes the area of S^{n-2} , $\alpha_{(p)}$ is the Lovelock coupling constant, and

$$\begin{aligned} \mathcal{L}_{(p)} = & \frac{(n-2)!}{(n-2p)!} \left[p \mathcal{R}[\bar{g}] R^{2-2p} + (n-2p)(n-2p-1) \left\{ (1-Z)^p + 2pZ \right\} R^{-2p} \right. \\ & \left. + p(n-2p) R^{1-2p} \left\{ 1 - (1-Z)^{p-1} \right\} (D_A R) \frac{(D^A Z)}{Z} \right]. \end{aligned} \quad (2.3)$$

Here $\mathcal{R}[\bar{g}]$ and D_A are the two-dimensional Ricci scalar and covariant derivative, respectively, and we have defined Z by the norm squared of the gradient of R , namely

$$Z := (DR)^2. \quad (2.4)$$

The generalized Misner-Sharp mass in Lovelock gravity was defined [28, 29] as

$$m_L := \frac{(n-2)\mathcal{A}_{(n-2)}}{16\pi G_{(n)}} \sum_{p=0}^{[n/2]} \tilde{\alpha}_{(p)} R^{n-1-2p} [1 - (DR)^2]^p, \quad (2.5)$$

where $\tilde{\alpha}_{(p)} := (n-3)!\alpha_{(p)}/(n-1-2p)!$. The gravitational field equations imply that $m_L = M$ is constant in vacuum, and yield the Schwarzschild-Tangherlini-type vacuum solution:

$$ds_{(n)}^2 = -f(R, M)dt^2 + f(R, M)^{-1}dR^2 + R^2 d\Omega_{(n-2)}^2. \quad (2.6)$$

Since $(DR)^2 = f(R, M)$ for the above metric, the form of $f(R, M)$ is determined algebraically from (2.5) [30].

We propose the following natural extension of (2.2) and (2.3):

$$I_{\text{XL}} = \frac{1}{l^{n-2}} \int d^2y \sqrt{-\bar{g}} \left\{ \phi(R) \mathcal{R}[\bar{g}] + \eta(R, Z) + \chi(R, Z) (D_A R) \frac{(D^A Z)}{Z} \right\}, \quad (2.7)$$

where $l^{n-2} := 16\pi G_{(n)}/\mathcal{A}_{(n-2)}$. Since the action (2.7) lives only in two spacetime dimensions, R and hence $Z := (DR)^2$ are scalars. The metric (but not the action in general) can be lifted to n dimensions by adding the angular piece $R^2 d\Omega_{(n-2)}^2$ as in (2.1), in which case R and Z recover their geometrical interpretations as areal radius and the norm squared of its gradient, respectively. The novel feature of the action (2.7) is that it contains arbitrarily high powers of $(DR)^2$, and hence the “velocity” $R_{,t}$, where a comma denotes the partial derivative. We will show in the next section that there exists a single integrability condition on the functions $\phi(R)$, $\eta(R, Z)$ and $\chi(R, Z)$ that guarantees the existence of a mass function in complete analogy with 2D dilaton gravity and EL gravity². This leaves sufficient flexibility in the choice of remaining functions in the theory to produce a large variety of interesting solutions.

3 Hamiltonian formalism and mass function

The Hamiltonian analysis of spherically symmetric EL gravity has been extensively studied [32, 33, 34, 25, 26]. The following is based in large part on the methodology and results

²We present only the results. Details will appear elsewhere [31].

of [25, 26] and uses the notation and conventions similar to [22]. We start with the general Arnowitt-Deser-Misner (ADM) metric in two spacetime dimensions:

$$ds^2 = -N^2 dt^2 + \Lambda^2 (N_r dt + dx)^2. \quad (3.1)$$

In this parametrization we have³

$$Z = -R_{,u}^2 + b^2, \quad (3.2)$$

where

$$b := \Lambda^{-2} R_{,x}, \quad (3.3)$$

$$R_{,u} := N^{-1} (R_{,t} - N_r R_{,x}). \quad (3.4)$$

In order to proceed with the Hamiltonian analysis, it is useful to assume that $\chi(R, Z)$ has an expansion of the form

$$\chi(R, Z) = \sum_I \beta^{(I)}(R) W_{(I)}(Z) Z. \quad (3.5)$$

In addition we assume that $W_{(I)}(Z) (= W_{(I)}(-R_{,u}^2 + b))$ has a Taylor expansion in Z and hence in $R_{,u}^2$, so that

$$W_{(I)}(Z) = \sum_m \frac{(-1)^m}{m!} \left. \frac{d^m W_{(I)}}{dZ^m} \right|_{Z=b} R_{,u}^{2m}. \quad (3.6)$$

A lengthy calculation [31] yields the total Hamiltonian density to be

$$\mathcal{H}_T = N\mathcal{H} + N_r \mathcal{H}_r, \quad (3.7)$$

with the Hamiltonian and diffeomorphism constraints given respectively by

$$\begin{aligned} \mathcal{H} = & P_R R_{,u} + \frac{1}{l^{n-2}} \left[2 \left(\frac{\phi_{,x}}{\Lambda} \right)_{,x} - \Lambda \eta(R, Z) \right. \\ & - 2R_{,u} \sum_I \left(\beta^{(I)}(R) W_{(I)}(Z) R_{,u} \frac{R_{,x}}{\Lambda} + 2\beta^{(I)}(R) \frac{R_{,x}}{\Lambda} \sum_m \frac{d}{db} \left(\frac{(-1)^m}{m!} \frac{d^m W_{(I)}}{dZ^m} \right)_{Z=b} \frac{R_{,u}^{2m+3}}{2m+3} \right)_{,x} \\ & \left. + 2\Lambda R_{,u} \sum_I \beta^{(I)}_{,R} \sum_m \frac{(-1)^m}{m!} \frac{d^m W_{(I)}}{dZ^m} \right|_{Z=b} \frac{R_{,u}^{2m+3}}{2m+3} - \sum_I \frac{R_{,x}}{\Lambda} \beta^{(I)}(R) W_{(I)}(Z) Z_{,x} \Big], \end{aligned} \quad (3.8)$$

$$\mathcal{H}_r = P_R R_{,x} - P_{\Lambda,x} \Lambda, \quad (3.9)$$

³Note that the definition of $R_{,u}$ departs slightly from the notation in [22] where y was used instead of u .

where the P_Λ and P_R are the conjugate momenta to the fields indicated by their subscripts. A crucial feature of the above is that $R_{,u}$ is a function of only P_Λ , Λ , and R , so that P_R appears only in the first term of each constraint above. One can therefore implement the general procedure for obtaining the mass function by taking the linear combination of \mathcal{H} and \mathcal{H}_r that eliminates P_R . After some algebra, one obtains the remarkably simple expression:

$$\begin{aligned}\tilde{\mathcal{H}} &:= \frac{R_{,x}}{\Lambda}\mathcal{H} - \frac{R_{,u}}{\Lambda}\mathcal{H}_r \\ &= (2\phi_{,RR}Z - \eta(R, Z))R_{,x} + (\phi_{,R} - \chi(R, Z))Z_{,x}.\end{aligned}\tag{3.10}$$

Equation (3.10) is a key result of our paper. It guarantees the existence of a mass function $\mathcal{M} = \mathcal{M}(R, Z)$ such that

$$\tilde{\mathcal{H}} = -\mathcal{M}_{,x}.\tag{3.11}$$

providing that the functions in the action satisfy the integrability condition:

$$(2\phi_{,RR}Z - \eta(R, Z))_{,Z} = (\phi_{,R} - \chi(R, Z))_{,R}\tag{3.12}$$

In this case, the total Hamiltonian can be written

$$H_T = \int dx \left(-\tilde{N}\mathcal{M}_{,x} + \tilde{N}_r\mathcal{H}_r \right) + H_B,\tag{3.13}$$

where the new Lagrange multipliers are

$$\tilde{N} := \frac{N\Lambda}{R_{,x}}, \quad \tilde{N}_r := N_r + N\frac{R_{,u}}{R_{,x}}.\tag{3.14}$$

Here H_B is the boundary term required to make the variational principle well defined. Assuming asymptotic flatness, $\tilde{N} \rightarrow 1$ holds at infinity, while we have $\mathcal{M} = M = \text{constant}$ for vacuum solutions. Thus the required boundary term is just the ADM mass:

$$H_B = \int dx \left(\tilde{N}\mathcal{M} \right)_{,x} \Big|_{x=x_B} = M,\tag{3.15}$$

where $x = x_B$ corresponds to the asymptotically flat region.

We note that the mass function \mathcal{M} commutes weakly with the constraints, and hence is a physical observable. It also commutes with the total Hamiltonian and is therefore constant not only in space but time as well.

4 Designer black holes

Given the existence of a mass function it is straightforward to derive the most general solution. One first needs to fix the diffeomorphism invariance. We choose $R = x$ and $P_\Lambda = 0$ to find a solution in the form of

$$ds^2 = -f(R, M)dt^2 + f(R, M)^{-1}dR^2. \quad (4.1)$$

Because $Z = 1/\Lambda^2$ in this gauge and $\mathcal{M}(R, Z) = \mathcal{M}(R, \Lambda^{-2})$ is equal to a constant, M , for vacuum solutions, the metric function $f(= \Lambda^2)$ is obtained by solving an algebraic equation $M = \mathcal{M}(R, f)$. The solution has at least one Killing vector, namely $\partial/\partial t$, and a single free parameter M . Up to possible degeneracies, the theory obeys a generalized Birkhoff theorem.

We now have the machinery to “design” an action to produce any given static 2D metric of the form (4.1). First invert the desired expression for $f = f(R, M)$ to write the mass M in terms of R and f . The result $M = \mathcal{M}(R, f)$ determines the covariant expression for the mass function $\mathcal{M} = \mathcal{M}(R, Z)$. One then determines the functions in the action by calculating

$$\frac{\partial \mathcal{M}}{\partial x} = \frac{\partial \mathcal{M}}{\partial R} \Big|_Z R_{,x} + \frac{\partial \mathcal{M}}{\partial Z} \Big|_R Z_{,x} \quad (4.2)$$

and comparing to (3.10) to identify two conditions that determine $\eta(R, Z)$ and $\chi(R, Z)$ keeping $\phi(R)$ arbitrary.

As a specific example, consider the Hayward metric (1.1). This leads to

$$\mathcal{M}(R, Z) = \frac{(1 - Z)R^{n-3}}{l^{n-2}} \left\{ 1 - (1 - Z)\frac{l^2}{R^2} \right\}^{-1}, \quad (4.3)$$

from which we derive the functions in the action as

$$\chi(R, Z) = \phi_{,R} - \frac{R^{n-3}}{2 \{1 - l^2(1 - Z)/R^2\}^2}, \quad (4.4)$$

$$\eta(R, Z) = 2\phi_{,RR}Z - \frac{R^{n-4}}{\{1 - l^2(1 - Z)/R^2\}^2} \left(1 - \frac{n-5}{2}(1 - Z)\frac{l^2}{R^2} \right). \quad (4.5)$$

Note that $\chi(R, Z)$ does have the power series assumed in the derivation.

There is a particularly interesting sub-class of theories, which we call *designer Lovelock gravity*. It corresponds to the dimensionally reduced action (2.2) for spherically symmetric EL, but with all the $\tilde{\alpha}_{(p)}$ potentially non-zero in n dimensions. In this case the action can no longer be lifted to a higher dimension EL theory since the corresponding Lovelock terms vanish algebraically for $p > n/2$. However, this provides us with an interesting 2D

generalization of the spherical theory that can be interpreted in one of two ways:

- (i) the large coupling limit $\alpha_{(p)} = \infty$ for $p \geq [(n-1)/2]$, or
- (ii) the large n limit ($n \rightarrow \infty$).

In designer Lovelock gravity, the metric function $f(R)$ is determined from a mass function that has an expansion of the form

$$\frac{2M}{R^{n-1}} = \sum_{p=0}^{\infty} \tilde{\alpha}_{(p)} \left(\frac{1-f(R)}{R^2} \right)^p. \quad (4.6)$$

Since the right-hand side is an infinite series it may be written as an analytic function of $(1-f)/R^2$ by choosing $\tilde{\alpha}_{(p)}$ appropriately. The Hayward black hole (1.1) is clearly in this class.

The following non-singular, asymptotically flat black hole is also realized in designer Lovelock gravity:

$$f(R) = 1 + \frac{R^{n+1}}{2l^{n+2}M} \left(1 - \sqrt{1 + \frac{4l^{2n}M^2}{R^{2(n-1)}}} \right). \quad (4.7)$$

The metric resembles the vacuum solution in Einstein-Gauss-Bonnet gravity [30, 35] but is realized with only odd-order Lovelock terms in the action.

On the other hand, another frequently studied non-singular black hole, the Bardeen-type black hole,

$$f(R) = 1 - \frac{l^{n-2}MR^2}{(R^2 + M^{2/(n-1)}l^{2n/(n-1)})^{(n-1)/2}}, \quad (4.8)$$

cannot be expressed as an infinite series of the form (4.6). In this case one must go to the full form of the action (2.7).

5 Conclusions

We have presented a new class of gravity theories in two spacetime dimensions that are readily understood as extensions of spherically symmetric Einstein-Lanczos-Lovelock gravity. They share many, if not all, of the latter's desirable properties: Second order equations and the existence of a mass function which in turn leads to a one parameter family of vacuum solutions with at least one Killing vector. In contrast to EL gravity, however, the extensions admit a large class of static non-singular black holes as vacuum solutions. By adding matter couplings and radiation back-reaction terms, one can study quantitatively the dynamics of the formation and evaporation of a larger class of interesting non-singular black holes than was previously available.

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